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D-branes in lorentzian AdS_3*

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Abstract: We study the exact construction of D-branes in lorentzian AdS_3. We start by defining a family of conformal field theories that gives a natural euclidean version of the $\mathrm{SL}(2,\mathbb{R})$ CFT and does not correspond to H_3^+, the analytic continuation of AdS_3. We argue that one can recuperate the exact CFT results of lorentzian AdS_3, upon an analytic continuation in the moduli space of these conformal field theories. Then we construct exact boundary states for various symmetric and symmetry-breaking D-branes in AdS_3.

Keywords: AdS-CFT and dS-CFT Correspondence, Conformal Field Models in String Theory, D-brane.

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Contents

1. Introduction and motivation

The three-dimensional anti-de-Sitter spacetime (AdS_3) has the unique virtue among other incarnations of the AdS/CFT holographic correspondence [1] of being solvable as a world-sheet theory. Indeed it corresponds to (the universal cover of) the WZW model for the group $\mathrm{SL}(2,\mathbb{R})$. Moreover it is a good laboratory to probe thermodynamical issues of gravity, since its black holes solutions are simply quotients by a discrete group, the famous BTZ black holes [2].

This exact conformal field theory is however difficult to solve, because the underlying group is non-compact. The correct spectrum itself has been found correctly only recently [3] (inspired by [4]) and confirmed later by a computation of the partition function of the $SL(2,\mathbb{R})$ WZW model [5]. Most of the other progress in this area has been made in the conformal field theory on the three-dimensional hyperbolic space H_3^+, i.e. the geometric coset $SL(2,\mathbb{C})/SU(2)$, obtained by an analytic continuation of AdS_3. The correlators of this theory have been computed through hard work in a series of papers [9, 8, 7, 6] using the chiral symmetries of the model. However it is quite difficult to extract the physics of the lorentzian AdS_3 from this conformal field theory, which is non unitary because of the imaginary NS-NS two-form. In particular its spectrum contains neither discrete representations nor long strings, which are distinguishing features of the $SL(2,\mathbb{R})$ WZW model. Nevertheless the correlators of $SL(2,\mathbb{R})$ have been obtained in [10] by taking this difficult route.

In this note, we argue that it is possible to give an alternative definition of euclidean AdS_3, which is more natural from the point of view of conformal field theory. We will study string theory on the following family of backgrounds:

$$\begin{aligned}
\mathrm{d}s^2 &= k\left[\mathrm{d}r^2 + \frac{R^2\tanh^2 r\ \mathrm{d}\varphi^2\ +\ \mathrm{d}x^2}{\tanh^2 r + R^2}\right] \\
\mathcal{B} &= \frac{k\ \tanh^2 r}{\tanh^2 r + R^2}\ \mathrm{d}x\wedge\mathrm{d}\varphi \\
e^{2\Phi} &= \frac{R^2\ e^{2\Phi_0}}{R^2\ \cosh^2 r + \sinh^2 r}
\end{aligned} \tag{1.1}$$

parameterized by the modulus R. Note that the NS-NS two-form is real and the dilaton bounded from above everywhere. As we shall see the corresponding CFT is unitary. This model captures most of the features of the $SL(2,\mathbb{R})$ conformal field theory; its spectrum contains the same discrete and continuous representations, and all sectors of spectral flow as well, which translates here into combinations of winding modes. However the geometry is rather different from AdS_3 since this manifold has only a $U(1)\times U(1)$ group of isometries and asymptotes a linear dilaton background. The AdS_3 background itself (the metric and the NS-NS two-form) is obtained for the value $R^2=-1$.

This family of theories is nothing but the the line of deformations of $SL(2,\mathbb{R})$ at level k by the truly marginal operator $\delta S = \int d^2z\ J^3\bar{J}^3$ that has been studied in [5], with an imaginary parameter though. This is indeed an exact CFT, T-dual to an orbifold of $SL(2,\mathbb{R})/U(1)|_A \times U(1)_{kR^2}$, where the first factor is the axially gauged WZW model, the well-known cigar CFT [11−13]. The issue of the analytic continuation in this description of the theory is then trivialized since the would-be timelike direction is simply a free bosonic field, coupled to the $SL(2,\mathbb{R})/U(1)$ model through its zero modes. This was (implicitly) the method used in [5] to compute the partition function of the $SL(2,\mathbb{R})$ WZW theory. The prescription that we shall employ to compute AdS_3 quantities is to do all the worldsheet computations in the theory defined by eq. (1.1) – which will be (analytic) functions of R – and at the end go back to the minkowskian AdS_3 theory through the continuation $R\to i$. This defines an *analytic continuation in the moduli space* of the conformal field theory.

One can easily check that the correlators computed in [10] can be obtained more simply using this prescription. Note that the analytic continuation in flat spacetime can be also formulated this way, and its extension to AdS_3 as we see is rather natural.

An essential ingredient to understand the physics of string theory in AdS_3 is to construct the D-branes in this spacetime. This is indeed the simplest setup to study D-branes in curved spacetimes and it has important implications for the holographic duality. These D-branes have been studied at the semi-classical level in [14, 15, 19]. However the exact boundary states are not known.[1] The main purpose of this work is to construct them, extending the procedure described above for the closed string theory to the boundary CFT. Indeed the D-branes in the H_3^+ theory has been constructed in [20, 21], and extended to the coset theory $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1)$ in [22] (see also [23 – 25]). We will use these results to construct the D-branes boundary states of the euclidean AdS_3 defined by eq. (1.1), and then in the lorentzian AdS_3 spacetime itself. Another advantage of this method is that it allows to construct the symmetry-breaking D-branes (see [26] for their analogues in the SU(2) WZW model) easily. Note finally that most of the expressions in this work will be valid not only for AdS_3 but also for the full line of $J^3\bar{J}^3$ marginal deformations of $\mathrm{SL}(2,\mathbb{R})$. In all the paper we will work in the bosonic string, although the extension to the supersymmetric case is rather simple.

This paper is organized as follows. We begin in section 2 by defining precisely the bulk theory of euclidean AdS_3. Then in the following sections we construct various class of D-branes. First we study in detail the much important AdS_2 D-branes in section 3, as well as their symmetry-breaking counterparts. Then we move to the localized D-branes in section 4, and D-branes covering AdS_3 in section 5. In the appendix we recall some facts about $\mathrm{SL}(2,\mathbb{R})$ representations and characters that will be used extensively in this work.

2. Euclidean AdS_3: closed string sector

The N-th cover of the WZW model $\mathrm{SL}(2,\mathbb{R})$ at level k corresponds to a timelike compactification of AdS_3 spacetime, with a non-zero NS-NS electric flux:

$$\mathrm{d}s^2 = k\left[\mathrm{d}r^2 + \sinh^2 r\, \mathrm{d}\varphi^2 - N^2\cosh^2 r\, \mathrm{d}x^2\right] \tag{2.1}$$

$$\mathcal{B} = Nk\ \sinh^2 r\ \mathrm{d}\varphi \wedge \mathrm{d}x \tag{2.2}$$

where we have normalized the time coordinate such that $x \sim x + 2\pi$. The physically sensible case corresponds to the *infinite cover* of the group $\mathrm{SL}(2,\mathbb{R})$ (i.e. $N \to \infty$), for which the globally defined time x is non-compact. However it will be convenient in the following to keep N arbitrary. After a T-duality along the time-like direction x we obtain the following torsionless solution with a non-trivial dilaton:

$$\mathrm{d}s^2 = k\left[\mathrm{d}r^2 + \tanh^2 r\left(\mathrm{d}\varphi + \frac{\mathrm{d}x}{kN}\right)^2 - \left(\frac{\mathrm{d}x}{kN}\right)^2\right] \tag{2.3}$$

$$\Phi = \Phi_0 - \log\cosh r\,. \tag{2.4}$$

[1] See [16 – 18] for previous works on this topic.

And after the redefinitions $\varphi + \frac{x}{kN} = \phi$ and $t = \frac{x}{kN}$ we get the conformal field theory $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1) \times \mathrm{U}(1)$, but with the identifications

$$(t,\phi) \sim \left(t + \frac{2\pi n}{kN}, \phi + \frac{2\pi n}{kN} + 2\pi m \right).$$

In other words this is the orbifold

$$\frac{\mathrm{SL}(2,\mathbb{R})_k/\mathrm{U}(1) \times \mathrm{U}(1)_{-k}}{\mathbb{Z}_{Nk}} \tag{2.5}$$

where $\mathrm{U}(1)_{-k}$ means a compact time-like U(1) of radius $\sqrt{2k}$.[2] the action of the orbifold is taken to be diagonal in the product CFT. A very important point is, while for the universal cover this procedure is well defined for an arbitrary level k (because we get a continuous orbifold), the theory can be properly defined for an arbitrary cover only if the level k is integer (this can be generalized to rational k). In the following we will mostly deal with the case k integer.

In this T-dual definition of the AdS_3 string theory the analytic continuation to euclidean space is rather trivial since the timelike direction is a free, compact boson coupled to the coset $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1)$ only through the action of the shift orbifold on its zero modes. This can be generalized slightly by starting with the following T-dual model

$$\begin{aligned} \mathrm{d}s^2 &= k \left[\mathrm{d}r^2 + \tanh^2 r \left(\mathrm{d}\varphi + \frac{\mathrm{d}x}{kN} \right)^2 + R^2 \left(\frac{\mathrm{d}x}{kN} \right)^2 \right] \\ \Phi &= \Phi_0 - \log\cosh r \end{aligned} \tag{2.6}$$

with still the periodicity $x \sim x + 2\pi$. It corresponds simply to changing the radius of the now spacelike direction x. After T-dualizing back it gives the following solution:

$$\mathrm{d}s^2 = k \left[\mathrm{d}r^2 + \frac{R^2 \tanh^2 r \, \mathrm{d}\varphi^2 + N^2 \mathrm{d}x^2}{\tanh^2 r + R^2} \right] \tag{2.7}$$

$$\begin{aligned} \mathcal{B} &= \frac{Nk \, \tanh^2 r}{\tanh^2 r + R^2} \, \mathrm{d}x \wedge \mathrm{d}\varphi \\ e^{2\Phi} &= \frac{R^2 \, e^{2\Phi_0}}{R^2 \, \cosh^2 r + \sinh^2 r} \end{aligned} \tag{2.8}$$

which is exactly the euclidean AdS_3 space given in eq (1.1). In the limiting cases $R \to 0$ and $R \to \infty$ one obtains respectively the vector coset $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1)$ – the trumpet – times a decoupled non-compact U(1), and the axial coset $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1)$ – the cigar.[3] This solution can also be obtained directly by considering a coset $[\mathrm{SL}(2,\mathbb{R}) \times \mathrm{U}(1)]/\mathrm{U}(1)$, much as in [31] for SU(2). Note finally that this background (and its T-dual) is very much related to the background of NS5-branes of a circle studied in [27, 28] (see also [29, 30]).[4] As a consequence our study of D-branes will be very close to the analysis of D-branes in this NS5-brane background performed in [32].

[2] Here and in the following we work with $\alpha' = 2$.

[3] More precisely, starting with the N-th cover of AdS_3 this background interpolates between the N-th cover of the trumpet and the cigar (which is uniquely defined), see [5] for details.

[4] The relation is indeed very accurate, since each section of the NS5-brane geometry for constant θ (see eq. (5.7) in [28]) gives such a background with the identification $R = \text{cotan}\ \theta$.

To close this discussion, let us recall that our definition of lorentzian AdS_3 will be given by the following analytic continuation:

$$\frac{\mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1)_k \times \mathrm{U}(1)_{R^2 k}}{\mathbb{Z}_{Nk}} \xrightarrow{R^2 \to -1} \mathrm{SL}(2,\mathbb{R})_k \tag{2.9}$$

and to get the physically desirable universal cover we take $N \to \infty$.

2.1 Closed string spectrum

To construct the closed string spectrum of the theory, we start with the partition function of $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1) \times \mathrm{U}(1)$ and implement the diagonal orbifold action in the standard way compatible with modular invariance. The partition function of the coset $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1)$ has been derived in [33] by means of a direct path integral approach (again, by using the H_3^+ results) and further analyzed in [28]. It is built with the coset characters that are recalled in the appendix. First we have a spectrum of discrete representations, of real spin in the range $\frac{1}{2} < j < \frac{k-1}{2}$ given by:

$$Z_{\mathrm{dis}}(\tau) = \sum_{n,w\in\mathbb{Z}} \int_{\frac{1}{2}}^{\frac{k-1}{2}} \mathrm{d}j \sum_{r,\bar{r}\in\mathbb{Z}} \delta(2j + r + \bar{r} + kw)\delta_{r-\bar{r},n} \lambda^d_{j,r}(\tau)\bar{\lambda}^d_{j,\bar{r}}(\bar{\tau}) \tag{2.10}$$

in terms of coset characters $\lambda^d_{j,r}(\tau)$ of the discrete representations. The constraint enforced by the delta-function comes from the coset construction. Note that this expression is valid for arbitrary k (non-integer or integer). In the k integer case, it can be recast in a more standard form since only half-integer values of the spin j appear. Let us recall that the spectrum of primary states reads:

$$\begin{aligned} L_0 &= -\frac{j(j-1)}{k-2} + \frac{(n+kw)^2}{4k} \\ \bar{L}_0 &= -\frac{j(j-1)}{k-2} + \frac{(n-kw)^2}{4k}\,. \end{aligned} \tag{2.11}$$

The second part of the spectrum is made with continuous representations, of imaginary spin $j = \frac{1}{2} + iP$. They correspond to asymptotic states that propagate in the cigar geometry, and consequently their spectrum is associated with an infinite volume divergence. This divergence can be regularized (much as in [34]), and leads to the result:

$$Z_{\mathrm{cont}}(\tau) = \int_0^\infty \mathrm{d}P \sum_{n,w\in\mathbb{Z}} \rho(P, n+kw, n-kw) \lambda^c_{\frac{1}{2}+iP,\frac{n+kw}{2}}(\tau)\, \bar{\lambda}^c_{\frac{1}{2}+iP,\frac{n-kw}{2}}(\bar{\tau}) \tag{2.12}$$

in terms of the characters $\lambda^c_{\frac{1}{2}+iP,m}$ of the continuous representations. The density of states ρ contains a leading part proportional to the (regularized) infinite volume of the manifold, and a subleading term given by the phase shift of the wave functions by the effective potential near the tip of the cigar [33]. The partition function contains also other non-universal contribution which appears because this infrared regulator breaks the chiral symmetries of the theory [28].

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The $\mathrm{U}(1)_{R^2k}$ part of the theory is rather trivial, since it is simply a free boson. Its spectrum if given by the following partition function:

$$Z_{\mathrm{u}(1)} = \frac{1}{\eta(\tau)\bar{\eta}(\bar{\tau})} \sum_{M,W\in\mathbb{Z}} q^{\frac{(M+kR^2W)^2}{4kR^2}} \bar{q}^{\frac{(M-kR^2W)^2}{4kR^2}} \tag{2.13}$$

with $q = \exp 2i\pi\tau$.

Let us move now to the orbifold theory that defines the euclidean AdS_3 CFT. In the closed string spectrum, the $\mathbb{Z}_{kN}$ orbifold has only an effect on the part of the $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ spectrum corresponding to the momenta of the compact $\mathrm{U}(1)$ in the asymptotic cylinder geometry (see eq. (2.11)), and the zero modes of the free $\mathrm{U}(1)_{R^2k}$ theory. The modification on the left and right momentum modes is as follows:

$$\frac{(n+kw, n-kw)}{\sqrt{2k}} \to \frac{\left(n+kw-\frac{\gamma}{N}, n-kw+\frac{\gamma}{N}\right)}{\sqrt{2k}}, \qquad \gamma \in \mathbb{Z}_{kN}$$

$$\frac{\left(M+R^2kW, M-R^2kW\right)}{R\sqrt{2k}} \to \frac{\left(n+kNp+R^2kW+\frac{R^2\gamma}{N}, n+kN\,p-R^2kW-\frac{R^2\gamma}{N}\right)}{R\sqrt{2k}}, \qquad p \in \mathbb{Z}\,.$$

Indeed invariance under the orbifold action enforces the constraint $M = n \mod Nk$, and twisted sectors labeled by $\gamma \in \mathbb{Z}_{kN}$ are requested for the modular invariance of the one-loop torus amplitude. Taking $N \to \infty$ and the analytic continuation $R \to i$, one can check easily that one obtains the partition function of the $\mathrm{SL}(2,\mathbb{R})$ WZW model computed in [5]. Trading the discrete momentum $\frac{\gamma}{kN}$ with the continuous parameter $s \in [0,1)$ in the $N \to \infty$ limit, the identification with the $\mathrm{SL}(2,\mathbb{R})$ quantum numbers is as follows:

$$m = -\frac{n}{2} + \frac{k}{2}(W+s)\,, \qquad \bar{m} = \frac{n}{2} + \frac{k}{2}(W+s)\,. \tag{2.14}$$

The combined winding number $W + w$ is identified with the sector of *spectral flow* of the $\widehat{\mathfrak{sl}}(2,\mathbb{R})$ representation (see [5] and the appendix), so the spectrum of the $\mathrm{SL}(2,\mathbb{R})$ CFT contains all the images under spectral flow of the discrete and continuous spectra. The spacetime energy of the state is given by $E = k(W+s)$, which can take any real value.

2.2 Bulk two-point function

To fix the normalization of the vertex operators and compare with the open string case, we will now give the two-point function for closed strings vertex operators. We shall denote in the following the vertex operators of the orbifoldized theory as:

$$V^j_{nw\gamma pW}(z,\bar{z}) = \Phi^{\mathrm{sl}(2)/\mathrm{u}(1)}_{j,n,w-\frac{\gamma}{kN}}(z,\bar{z})\,\Phi^{\mathrm{u}(1)}_{n+kNp,W+\frac{\gamma}{kN}}(z,\bar{z})\,. \tag{2.15}$$

Starting from the two-point function of the coset $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ computed in [30, 35], the two-point function on the sphere is given as follows:

$$\begin{aligned}\langle V^j_{nw\gamma pW}(z_1,\bar{z}_1) V^{j'}_{n'w'\gamma'p'W'}(z_2,\bar{z}_2)\rangle &= |z_2-z_1|^{-4\Delta_{jnw\gamma pW}} \\ &\quad \times\delta_{n,-n'}\delta_{kNw-\gamma,-kNw'+\gamma'}\delta_{p,-p'}\delta_{W,-W'} \times\end{aligned} \tag{2.16}$$

$$\times\left[\delta(j+j'+1)+R\left(j,n,w-\frac{\gamma}{kN}\right)\delta(j-j')\right]$$

with the reflection amplitude

$$R\left(j,n,w-\frac{\gamma}{kN}\right)=\nu^{1-2j}\frac{\Gamma(1-2j)\Gamma(1+\frac{1-2j}{k-2})}{\Gamma(2j-1)\Gamma(1+\frac{2j-1}{k-2})}\frac{\Gamma(j+\frac{n+kw-\frac{\gamma}{N}}{2})\Gamma(j+\frac{n-kw+\frac{\gamma}{N}}{2})}{\Gamma(1-j+\frac{n+kw-\frac{\gamma}{N}}{2})\Gamma(1-j+\frac{n-kw+\frac{\gamma}{N}}{2})} \tag{2.17}$$

and the normalization constant $\nu=\Gamma(1-\frac{1}{k-2})/\Gamma(1+\frac{1}{k-2})$. By taking the limit of the universal cover $N\to\infty$ and identifying the $\mathrm{SL}(2,\mathbb{R})$ quantum numbers as (2.14) we obtain the two-point function of lorentzian AdS_3 given in [10]. This can be easily generalized to three-point functions. Note that the non-trivial part of the correlators is independent of the parameter R. This concludes our discussion of the bulk theory. We now move to the boundary theory in the following sections.

3. AdS_2 D-branes

We start the discussion of the boundary conformal field theory with the construction of AdS_2 branes in the lorentzian AdS_3. These D-branes were found to be associated with *twined conjugacy classes* of $\mathrm{SL}(2,\mathbb{R})$ [14]. They are certainly the more interesting D-branes to consider since they have a well defined holographic interpretation; they correspond to domain walls in the spacetime CFT dual to AdS_3 [36]. The boundary states for the associated D-branes in H_3^+ have been found in [20, 21] by solving factorization constraints involving degenerate operators. Our aim is to construct them in lorentzian AdS_3, by starting with the euclidean AdS_3 CFT defined above. We will start with the appropriate D-branes in $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)\times\mathrm{U}(1)$ and move to the orbifold theory.

The AdS_2 D-brane of $\mathrm{SL}(2,\mathbb{R})$ descends in the coset theory $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ of the cigar to infinite D1-branes of embedding equation:

$$\sinh r_0=\sinh r\sin(\varphi-\varphi_0)\,. \tag{3.1}$$

The parameter φ_0 corresponds to the position of this D1-brane on the compact circle at infinity (since the cigar asymptotes $\mathbb{R}_{\mathcal{Q}}\times S^1$) and the parameter r_0 gives the distance of the D-brane to the tip of the cigar $r=0$. The exact one-point function on the disk corresponding to this D1-brane has been obtained from descent of H_3^+ as [20, 22]:[5]

$$\begin{aligned}\langle\Phi^{\mathrm{sl}(2)/\mathrm{u}(1)}_{j,nw}\rangle_{r_0,\varphi_0}=\;&\delta_{w,0}\frac{1}{\sqrt{2}}\left(\frac{k-2}{k}\right)^{1/4}\nu^{\frac12-j}\,e^{in\varphi_0}\times\\&\times\left[e^{-r_0(1-2j)}+(-1)^n e^{r_0(1-2j)}\right]\frac{\Gamma(1-2j)\Gamma(1+\frac{1-2j}{k-2})}{\Gamma(1-j+\frac{n}{2})\Gamma(1-j-\frac{n}{2})}\,.\end{aligned} \tag{3.2}$$

This expression contains only coupling of closed string modes belonging to the continuous representations, for $j=\frac12+iP$. There are no poles associated to discrete representations of $\mathrm{SL}(2,\mathbb{R})$ that would signal the coupling of the D-brane to the corresponding closed string

[5] Here and in the following we have suppressed the $(z,\bar z)$ dependence. One should read $\langle V_{\alpha,\bar\alpha}(z,\bar z)\rangle_{\hat\beta}=|z-\bar z|^{-\Delta_\alpha-\Delta_{\bar\alpha}}\Psi_{\hat\beta}(\alpha,\bar\alpha)$ and the coefficient $\Psi_{\hat\beta}(\alpha,\bar\alpha)$, including the selection rules, is given in the text.

states. However the gamma-functions in the numerator give poles associated to the infinite volume of the target space (they are "bulk poles" using the classification of [37]).

To obtain a AdS_2 D-brane we have to choose for the extra $\mathrm{U}(1)_{R^2k}$ theory Dirichlet boundary conditions (otherwise we obtain the symmetry-breaking D-branes constructed at the end of this section). The corresponding one-point function is simply:

$$\langle \Phi^{\mathrm{u}(1)}_{M,W} \rangle_{x_0} = \delta_{W,0} \, \frac{e^{iMx_0}}{(\sqrt{2k}R)^{1/2}} \tag{3.3}$$

for a D-brane located at $x = x_0$. Now let us move to the (euclidean) AdS_3 CFT.

3.1 Boundary states for AdS_2 branes in AdS_3

In the geometry (2.4) of the orbifold theory we start with a D-brane of embedding equation:

$$\sinh r_0 = \sinh r \sin\left(\varphi + \frac{x}{kN} - \varphi_0\right) , \qquad \frac{x}{kN} = x_0 \tag{3.4}$$

corresponding to a D1-brane of the cigar and a D0-brane of U(1) as discussed above. After a T-duality along x we obtain the profile of the D-brane upon eliminating the dualized coordinate x, which is then identified with the gauge field. This general technique is discussed in [38] and has been heavily used in [32] for the case of D-branes in NS5-geometries, which is very much related to our problem as we already discussed. It gives a D2-brane in (euclidean) AdS_3 of profile:

$$\sinh r_0 \ = \sinh r \sin(\varphi + x_0 - \varphi_0) \, . \tag{3.5}$$

The parameter x_0 can be absorbed in a redefinition of the (continuous) parameter φ_0. It is interesting to remark that the embedding equation and the gauge field are the same for all the geometries (1.1), irrespectively of the actual value of the parameter R. In the lorentzian AdS_3 geometry ($R^2 = -1$), the embedding equation of the D-brane defines an AdS_2 submanifold of AdS_3.

Note that the gauge for the B-field is fixed implicitly by the particular T-duality we use for the construction of the D-branes. In this gauge the magnetic field on the D-branes vanishes. However, we can choose instead the gauge

$$\mathcal{B}' = Nk \ \cosh^2 r \ \mathrm{d}x \wedge \mathrm{d}\varphi \tag{3.6}$$

for the NS-NS two-form in AdS_3. Then the T-dual background will be given by

$$\mathrm{d}s^2 = k \left[\mathrm{d}r^2 + \tanh^2 r \left(\frac{\mathrm{d}x}{kN}\right)^2 - \left(\mathrm{d}\varphi - \frac{\mathrm{d}x}{kN}\right)^2 \right] , \tag{3.7}$$

giving by the same reasoning the following magnetic field on the D-brane:

$$F = \frac{kN}{2\pi} \, \mathrm{d}\varphi \wedge \mathrm{d}x = -\frac{kN}{2\pi} \, \frac{\sinh r_0 \coth r}{\sqrt{\sinh^2 r - \sinh^2 r_0}} \, \mathrm{d}r \wedge \mathrm{d}x \, , \tag{3.8}$$

the two expressions being related by the embedding equation.

Let us now consider the exact CFT description of those D-branes. To construct the D-branes in the $\mathbb{Z}_{kN}$ diagonal orbifold CFT we have to sum over the images $(\varphi_0+\frac{2\pi\ell}{kN}, x_0-\frac{2\pi\ell}{kN})$, $\ell\in\mathbb{Z}_{kN}$. It will impose $M=n \mod kN$ which is the condition of invariance of the closed string state under the orbifold action. Using the labeling of the primary states given by eq (2.15), we obtain the following one-point function for an AdS$_2$ D-brane of parameters (r_0,φ_0,x_0):

$$\begin{aligned}\langle V^j_{nw\gamma pW}\rangle_{r,\varphi_0} &= \delta_{Nk\,w-\gamma,0}\;\delta_{Nk\,W+\gamma,0}\,\nu^{\frac{1}{2}-j}\left(\frac{k-2}{2}\right)^{1/4}\frac{1}{\sqrt{2R}}e^{in\,(\varphi_0+x_0)}\;e^{ikNp\,x_0}\times\\ &\quad\times\left[e^{-r_0(1-2j)}+(-1)^n e^{r_0(1-2j)}\right]\frac{\Gamma(1-2j)\Gamma(1+\frac{1-2j}{k-2})}{\Gamma(1-j+\frac{n}{2})\Gamma(1-j-\frac{n}{2})}\,.\end{aligned}\tag{3.9}$$

This expression is valid for all the family of euclidean AdS$_3$ solutions of eq. (1.1). It gives also the boundary state for the AdS$_2$ D-brane in lorentzian AdS$_3$ by taking the analytic continuation $R\to i$.

3.2 Annulus amplitude

To compute the spectrum of open strings (and check the consistency of this D-brane), we will use the channel duality of the annulus amplitude [39]. We start by writing the annulus amplitude in the closed string channel, which is obtained from the closed string spectrum of continuous representations in the orbifold theory and the overlap of the boundary states whose coefficients are given by the one-point function (3.9). Since this boundary states contains only Ishibashi states associated to continuous representations (as in SL$(2,\mathbb{R})/$U(1)) we obtain the following amplitude between two D-branes of parameters (r_0,φ_0,x_0) and (r_0',φ_0',x_0'), with $\tilde\tau=-1/\tau$:

$$\begin{aligned}Z^{\text{CL}}(\tilde\tau) &= \frac{1}{2\sqrt{2(k-2)}R}\sum_{n,p\in\mathbb{Z}} e^{i(n+kNp)(x_0'-x_0)}\frac{\tilde q^{\frac{(n+kNp)^2}{4kR^2}}}{\eta(\tilde\tau)}\int_0^\infty \mathrm{d}P\;\frac{\cosh 2\pi P+(-)^n}{\sinh 2\pi P\sinh\frac{2\pi P}{k-2}}\times\\ &\quad\times e^{i(\varphi_0'-\varphi_0)n}\left\{\cos 2P(r_0-r_0')+(-)^n\cos 2P(r_0+r_0')\right\}\lambda^c_{\frac{1}{2}+iP,\frac{n}{2}}(\tilde\tau)\,.\end{aligned}\tag{3.10}$$

Then we can modular transform to the open string channel, using the formulas discussed in the appendix. There is a divergence associated to the infinite volume available to the open strings. To remove this universal divergence from the amplitude we construct the *relative partition function* w.r.t. to a spectrum of open string stretched between reference D-branes of parameters (r_*,φ_0,x_0) and (r_*',φ_0',x_0'), see [20]. We obtain the following one-loop amplitude for the open strings stretched between to AdS$_2$ branes in euclidean AdS$_3$:

$$\begin{aligned}&Z^{\text{OP}}_{(r_0,\varphi_0,x_0;r_0',\varphi_0',x_0')}(\tau)-\\ &-Z^{\text{OP}}_{(r_*,\varphi_0,x_0;r_*',\varphi_0',x_0')}(\tau) = \frac{1}{N}\sum_{m,w\in\mathbb{Z}}\frac{1}{\eta(\tau)}\,q^{\frac{R^2}{k}\left(\frac{m}{N}+\frac{k(x_0'-x_0)}{2\pi}\right)^2}\times\\ &\qquad\times\sum_{W\in\mathbb{Z}}\int \mathrm{d}P'\left\{\frac{\partial\log\frac{R(P'|r_0,r_0')}{R(P'|r_*,r_*')}}{2i\pi\partial P'}\lambda^c_{P',-\frac{m}{N}+kW+\frac{k(\varphi_0'-\varphi_0)}{2\pi}}(\tau)+\right.\end{aligned}\tag{3.11}$$

$$
+\frac{\partial \log \frac{R(P'|r_0,-r_0')}{R(P'|r_*,-r_*')}}{2i\pi\partial P'}\,\lambda^c_{P',-\frac{m}{N}+k(W+\frac{1}{2})+\frac{k(\varphi_0'-\varphi_0)}{2\pi}}(\tau)\Bigg\}\,.
$$

The first and second term contain integer and half-integer windings respectively. Note that the normalization of the open string partition function has be chosen to be compatible with the limit of the infinite cover (continuous orbifold), see below. This result is expressed in terms of the reflection amplitude for open strings streched two AdS_2 D-branes (r, r') in H_3^+, whose (r, r')-dependent part reads [20, 40]:

$$
R(P|r,r') = \frac{S_k^{(0)}\left(\frac{k-2}{2\pi}(r+r')+P\right)}{S_k^{(0)}\left(\frac{k-2}{2\pi}(r+r')-P\right)}\frac{S_k^{(1)}\left(\frac{k-2}{2\pi}(r-r')+P\right)}{S_k^{(1)}\left(\frac{k-2}{2\pi}(r-r')-P\right)}\,, \tag{3.12}
$$

in terms of the special functions:

$$
\log S_k^{(0)}(x) = i\int_0^\infty \frac{dt}{t}\left(\frac{\sin\frac{2tx}{k-2}}{2\sinh t\,\sinh\frac{t}{k-2}} - \frac{x}{t}\right) \tag{3.13}
$$

$$
\log S_k^{(1)}(x) = i\int_0^\infty \frac{dt}{t}\left(\frac{\cosh t\,\sin\frac{2tx}{k-2}}{2\sinh t\,\sinh\frac{t}{k-2}} - \frac{x}{t}\right). \tag{3.14}
$$

We discuss this reflection amplitude in more detail now.

3.3 The boundary two-point function

As in the bulk the reflection amplitude comes from the two-point function of boundary operators:

$$
\langle \Phi^{rr'}_{j,\mu}(x)\Phi^{rr'}_{j',-\mu}(y)\rangle = |x-y|^{-2\Delta_{j,\mu}}\delta(j-j')\;R(j,\mu|r,r') \tag{3.15}
$$

with

$$
\mu = -\frac{m}{N} + kW + \frac{k(\varphi_0'-\varphi_0)}{2\pi}
$$

or

$$
\mu = -\frac{m}{N} + k\left(W+\frac{1}{2}\right) + \frac{k(\varphi_0'-\varphi_0)}{2\pi}\,.
$$

The full reflection amplitude for the AdS_2 D-branes in H_3^+ has been computed in [20] for open strings with both ends on the same D-brane and in [40] in the generic case. We can then obtain from descent the corresponding reflection amplitude in the coset $SL(2,\mathbb{R})/U(1)$ (by means a Fourier transform much as in [30] and explained in the appendix) and then lift the result to the euclidean AdS_3 by means of the orbifold construction. Then the full expression for the reflection amplitude reads:

$$
\begin{aligned}
&R(j,\mu|r,r') = \\
&= \nu^{\frac{1}{2}-j}\frac{\Gamma_k^2(k-2+j)}{\Gamma_k^2(k-2+1-j)}\frac{\Gamma_k(k-2+1-2j)}{\Gamma_k(k-2+2j-1)}\times \\
&\times \frac{S_k^{(0)}\left(\frac{k-2}{2\pi}(r+r')-i(j-1/2)\right)}{S_k^{(0)}\left(\frac{k-2}{2\pi}(r+r')+i(j-1/2)\right)}\frac{S_k^{(1)}\left(\frac{k-2}{2\pi}(r-r')-i(j-1/2)\right)}{S_k^{(1)}\left(\frac{k-2}{2\pi}(r-r')+i(j-1/2)\right)}\frac{S_k^{(0)}(+i(j-1/2))}{S_k^{(0)}(-i(j-1/2))}\times
\end{aligned}
$$

$$\times \frac{\Gamma(j-\mu)}{\Gamma(1-j-\mu)} \tag{3.16}$$

where the last factor comes from the Fourier transform, see appendix. This expression involves the non-trivial function

$$\Gamma_k(x) = (k-2)^{-\frac{x(x+2-k)}{2(k-2)}}(2\pi)^{x/2}\Gamma_2^{-1}(x|1,k-2) \tag{3.17}$$

written in terms of $\Gamma_2(x|\omega_1,\omega_2)$ the Barnes double gamma-function.

This expression has indeed poles signaling the presence of discrete representations, for $\mu - j \in \mathbb{N}$. Note that for the coset $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ ($\mu = kW$) there are also poles of this kind provided k is non-integer.[6] This seems to contradict the fact that no discrete representations appear in the open string spectrum (3.11), and the related observation that the one-point function (3.9) does not contain poles corresponding to couplings of closed string modes in the discrete representations with the D-brane. To solve this puzzle, one can invoke the fact (this was suggested in [22] for the corresponding brane in the coset) that what we really compute is a relative partition function, and the open string spectrum is expected not to depend on the parameters r_0 and r_0' of the D-branes. Indeed semi-classical analysis [15] and the computation of the free energy [21] suggest that the open string spectrum contains a full discrete spectrum with all allowed values in the range $\frac{1}{2} < j < \frac{k-1}{2}$ – much as the closed string spectrum.[7]

3.4 Open string spectrum in lorentzian spacetime

Now let us go discuss the open string spectrum on the AdS_2 D-branes in lorentzian AdS_3. As we already discussed in detail we have first to take the limit $N \to \infty$ in (3.11) to go to the universal cover of the $\mathrm{SL}(2,\mathbb{R})$ group manifold. We have to replace the sum $\frac{1}{N}\sum_m f(m/N)$ by the Riemann integral $\int dE\ f(E)$ (and we can absorb x_0 by a shift of φ_0). Then we can make the analytic continuation $R \to i$ to go back to lorentzian AdS_3. To express the result in terms of $\mathrm{SL}(2,\mathbb{R})$ characters we have to choose $\varphi_0' = \varphi_0 \mod 2\pi$. This is indeed necessary in order that the D-brane configuration preserves an $\mathrm{SL}(2,\mathbb{R})$ symmetry. Then we find the following open string partition function on the AdS_2 branes:

$$Z^{\mathrm{OP}}_{\mathrm{AdS}_2}(\tau) = \int dE \sum_{w\in\mathbb{Z}} \int \mathrm{d}P' \left\{ \frac{\partial \log \frac{R(P'|r_0,r_0')}{R(P'|r_*,r_*')}}{2i\pi\partial P'} \chi^{c\,(2w)}_{\frac{1}{2}+iP',E}(\tau) + \right.$$
$$\left. + \frac{\partial \log \frac{R(P'|r_0,-r_0')}{R(P'|r_*,-r_*')}}{2i\pi\partial P'} \chi^{c\,(2w+1)}_{\frac{1}{2}+iP',E}(\tau) \right\} \tag{3.18}$$

written in terms of the $\mathrm{SL}(2,\mathbb{R})$ characters $\chi^{c\,(w)}_{\frac{1}{2}+iP',m}$ in the w-flowed representation. They are obtained from the coset characters λ^c_m and the U(1) characters with the branching relations given in the appendix. This gives the following open string spectrum

$$L_0 = \frac{P^2+\frac{1}{4}}{k-2} - 2\left(w+\frac{\epsilon}{2}\right)E + k\left(w+\frac{\epsilon}{2}\right)^2, \qquad \epsilon = 0,1\,. \tag{3.19}$$

[6]Indeed, using the range allowed for j, we find that the denominator is negative, and integer for k integer.

[7]I thank Jan Troost for discussions on this point.

This is exactly what we expect for continuous representations with $m = E$ and $(2w+\epsilon)$ units of spectral flow. The open strings with even spectral flow have both ends at the same point on the D-brane, and those with odd spectral flow end on different points (they correspond to half-integer windings). Thus the on-shell continuous spectrum on the AdS_2 D-branes consists in long strings, in agreement with the semi-classical analysis of [19].

3.5 Symmetry-breaking D-brane

As in SU(2) [26] we can construct "B-branes" preserving only a $\widehat{\mathfrak{u}(1)}$ affine subalgebra of $\widehat{\mathfrak{sl}(2,\mathbb{R})}$. We already stressed in the introduction that the decomposition of $\mathrm{SL}(2,\mathbb{R})$ as $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1) \times \mathrm{U}(1)$ is especially convenient to construct those D-branes.

The first category of such D-branes are the symmetry-breaking cousins of the AdS_2 D-branes. To obtain them we combine a D1-brane of the axial coset $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ (the cigar) with a Neumann D-brane of $\mathrm{U}(1)_{R^2k}$. Going through the same steps of T-duality, we find that these symmetry breaking D2-branes cover the region $r > r_0$ of AdS_3 and carry the following electric field, for instance in the first gauge choice (see also [41, 42]):

$$F = -\frac{kN}{2\pi}\left[\frac{\sinh r_0 \tanh r}{\sqrt{\sinh^2 r - \sinh^2 r_0}}\mathrm{d}r\wedge \mathrm{d}x + \mathrm{d}\phi \wedge \mathrm{d}x\right]. \tag{3.20}$$

Using a similar reasoning as for the AdS_2 branes, we find the following one-point function for the symmetry-breaking D-brane:

$$\begin{aligned}\langle V^j_{nw\gamma pW}\rangle_{r_0} &= \delta_{Nk\,w+\gamma,0}\delta_{n+kNp,0}\sqrt{\frac{R}{2}}\left(\frac{k-2}{2}\right)^{1/4}\nu^{1/2-j}\times\\ &\quad\times\left[e^{-r_0(1-2j)}+e^{i\pi kNp}\ e^{r_0(1-2j)}\right]\frac{\Gamma(1-2j)\Gamma(1+\frac{1-2j}{k-2})}{\Gamma(1-j+\frac{kNp}{2})\Gamma(1-j-\frac{kNp}{2})}.\end{aligned} \tag{3.21}$$

The only difference with the symmetric D-branes is that the boundary conditions along the two U(1) directions (the compact U(1) of the cigar and the extra $\mathrm{U}(1)_{R^2k}$) are different; indeed the annulus amplitude in the closed string channel will involve only U(1) characters with no momentum. Thus after a modular transform we get the following open string spectrum (for the universal cover of the lorentzian AdS_3):[8]

$$\begin{aligned}Z^{\mathrm{OP}}(\tau) = \int \mathrm{d}E\frac{1}{\eta(\tau)}q^{-\frac{E^2}{k}}\int_0^\infty \mathrm{d}P'\int d\mu\Bigg\{&\frac{\partial\log\frac{R(P'|r_0,r_0')}{R(P'|r_*,r_*')}}{2i\pi\partial P'}\lambda^c_{\frac{1}{2}+iP',\mu}(\tau)+\\ &+\frac{\partial\log\frac{R(P'|r_0,-r_0')}{R(P'|r_*,-r_*')}}{2i\pi\partial P'}\lambda^c_{\frac{1}{2}+iP',\mu+\frac{k}{2}}(\tau)\Bigg\}.\end{aligned} \tag{3.22}$$

Since this partition function cannot be expressed in terms of $\widehat{\mathfrak{sl}(2,\mathbb{R})}$ characters, it is clear that this D-brane breaks the $\widehat{\mathfrak{sl}(2,\mathbb{R})}$ symmetry of the CFT.

[8]Of course the two terms can be brought together but this is not the case for an arbitrary cover.

4. Localized D-branes

We now move to D-branes that are localized in space, at the origin $r = 0$ of global coordinates in AdS_3. In the H_3^+ CFT there is a series of D-branes corresponding to spheres of imaginary radius, labeled by an integer label $u = 1, 2, \cdots$. Even if their geometrical interpretation is not clear, they can be reduced to D-branes in the cigar CFT, corresponding to D0-branes sitting at the tip $r = 0$ [22]. However their open string spectrum is associated to a *finite* representation of $\widehat{\mathfrak{sl}(2,\mathbb{R})}$ of spin $j = -(u-1)/2$, which is generically not unitary [23]. Thus only the D-brane with $u = 1$ is physical because it corresponds to a trivial representation which is indeed unitary. Note also that this particular D-brane has a clear geometrical interpretation in H_3^+ as a point-like object. The associated one-point function for the D0-brane in the cigar is given by [22]:

$$\langle \Phi_{j\,nw} \rangle = \delta_{n,0} \left(\frac{k}{k-2} \right)^{1/4} \frac{\nu^{\frac{1}{2}-j}}{\sqrt{k-2}} (-)^w \frac{\Gamma(j + \frac{kw}{2})\Gamma(j - \frac{kw}{2})}{\Gamma(2j-1)\Gamma(1 + \frac{2j-1}{k-2})} \,. \tag{4.1}$$

This has been generalized in [23] to D0-branes in an orbifold that we will need afterwards.

4.1 Symmetric D-brane: D-instanton

The symmetric D-brane associated to the D0-brane in the cigar is a D-instanton in AdS_3 [14]. It is obtained by starting with the D0-brane of the cigar together with a Neumann D-brane for the extra U(1). More precisely, for the single cover of $\mathrm{SL}(2,\mathbb{R})$ we obtain two copies of the D-instanton (for $x = 0$ and $x = \pi$) associated to the center of $\mathrm{SL}(2,\mathbb{R})$ (see e.g. [42]), and for the N-th cover we have N copies of this picture, therefore N copies of the open string spectrum. Thus we will restrict our discussion to the single cover.

Following the same logic as before, we obtain the following one-point function for the D-instanton in AdS_3:

$$\langle V^j_{nw\gamma pW} \rangle = \frac{\sqrt{R}\,k}{2^{1/4}(k-2)^{3/4}}\, \delta_{n,0}\, \delta_{p,0}\, \nu^{\frac{1}{2}-j} \frac{\Gamma(j + \frac{kw-\gamma}{2})\Gamma(j - \frac{kw-\gamma}{2})}{\Gamma(2j-1)\Gamma(1 + \frac{2j-1}{k-2})} \,. \tag{4.2}$$

This one-point function contains both couplings to the closed string states of the continuous representations ($j = \frac{1}{2} + iP$) and of the discrete representations ($\frac{1}{2} < j < \frac{k-1}{2}$). To compute the contribution of the latter to the closed string annulus amplitude, we take simply the residue at the corresponding simple pole. Here for simplicity we will only give the continuous part of the closed string annulus amplitude (more details can be found in [22, 23]):

$$\begin{aligned} Z^{\text{CL}}(\tilde{\tau}) = {} & \frac{k^2 R}{\sqrt{2(k-2)}} \sum_{w,W\in\mathbb{Z}} \sum_{\gamma\in\mathbb{Z}_k} \times \\ & \times \int_0^\infty dP \frac{\sinh 2\pi P \sinh \frac{2\pi P}{k-2}}{\cosh 2\pi P + \cos \pi(kw-\gamma)} \frac{\tilde{q}^{\frac{kR^2}{4}(W+\gamma)^2}}{\eta(\tilde{\tau})} \lambda^c_{\frac{1}{2}+iP,\,kw-\gamma}(\tilde{\tau}) \,. \end{aligned} \tag{4.3}$$

The modular transformation from the open string channel to the closed string channel, giving the closed string channel annulus amplitude of the D0-brane (for both continuous and discrete states) in $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ is non-trivial and has been computed in [22, 23]. In our case the boundary state defined by (4.2) corresponds to the following open string partition function for the D(-1)-brane of euclidean AdS_3:

$$Z^{\mathrm{OP}}_{D(-1)}(\tau) = \sum_{r,w\in\mathbb{Z}} \lambda^{\mathbb{I}}_r(\tau) \frac{q^{\frac{(r+kw)^2}{kR^2}}}{\eta(\tau)} \tag{4.4}$$

such that for lorentzian AdS_3 we get simply the identity representation of $\mathrm{SL}(2,\mathbb{R})$ and its images under spectral flow, see the appendix.

4.2 Symmetry-breaking D-brane: D-particle

The symmetry breaking D-branes associated to these localized branes are D-particles sitting at the origin $r=0$. In this case one take a Dirichlet D-brane for the extra U(1). Then we obtain the following one-point function:

$$\langle V^j_{nw\gamma pW}\rangle = [2(k-2)]^{-1/4}\sqrt{\frac{k}{R(k-2)}}\delta_{n,0}\delta_{kNW+\gamma,0}e^{ix_0\,kNp}\nu^{\frac{1}{2}-j}\frac{\Gamma(j+\frac{kw}{2})\Gamma(j-\frac{kw}{2})}{\Gamma(2j-1)\Gamma(1+\frac{2j-1}{k-2})}\,. \tag{4.5}$$

It gives the following closed strings amplitude for the continuous representations:

$$\begin{aligned} Z^{\mathrm{CL}}(\tilde\tau) &= \frac{k}{R\sqrt{2(k-2)}}\sum_{w,p\in\mathbb{Z}}\times \\ &\times \int_0^\infty dP\, e^{i(x_0'-x_0)kNp}\frac{2\sinh 2\pi P\sinh\frac{2\pi P}{k-2}}{\cosh 2\pi P+\cos\pi kw}\frac{\tilde q^{\frac{kN^2}{4R^2}p^2}}{\eta(\tilde\tau)}\lambda^{c}_{\frac12+iP,kw}(\tilde\tau) \end{aligned} \tag{4.6}$$

with again a contribution of discrete representations that can be found in [22, 23] for the coset $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$. The computation is exactly similar since for these symmetry-breaking D-branes the cigar CFT and the extra U(1) CFT are decoupled. This annulus amplitude in the closed string channel is compatible with the following open string partition function:

$$Z^{\mathrm{OP}}_{D0}(\tau) = \frac{1}{N}\sum_{r\in\mathbb{Z}}\lambda^{\mathbb{I}}_r(\tau)\sum_{m\in\mathbb{Z}}\frac{q^{\frac{R^2}{k}\left(\frac{m}{N}-\frac{k(x_0'-x_0)}{2\pi}\right)^2}}{\eta(\tau)} \tag{4.7}$$

written in terms of the characters $\lambda^{\mathbb{I}}_r$ of the identity representation for the coset. In the universal cover of lorentzian AdS_3 we have simply

$$Z^{\mathrm{OP}}_{D0}(\tau) = \sum_{r}\lambda^{\mathbb{I}}_r(\tau)\int dE\;\frac{q^{-\frac{E^2}{k}}}{\eta(\tau)} \tag{4.8}$$

and the spectrum of zero modes is the same as for open strings attached to a D-particle in flat space.

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5. Extended D-branes

We consider here D-branes of AdS_3 that are constructed from the D2-branes of the cigar.

5.1 H_2 S-brane

These D2-branes are space-like D-branes in AdS_3. By descent to the coset theory one obtains a D2-brane of $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1)$ covering all the cigar, with a magnetic field:

$$F = \frac{k}{2\pi} \frac{\sin\sigma \tanh^2 r}{\sqrt{\cosh^2 r - \sin^2\sigma}} \mathrm{d}r \wedge \mathrm{d}\phi\,. \tag{5.1}$$

where $\sigma \in [0, \pi/2)$.[9] The one point function of this D2-brane has been computed in [22]. However, as shown in [23], they lead to an open string spectrum with negative multiplicities. It was further argued that such a problem disappears when the level k is integer [32]. We will restrict our discussion to this case for now. The one-point function for the D2-branes then reads [22]:

$$\begin{aligned} \langle \Phi^j_{nw} \rangle_\sigma \; = \; & \frac{[k(k-2)]^{1/4}}{2\pi(k-2)} \nu_k^{1/2-j}\, \delta_{n,0} \times \\ & \times \Gamma(2-2j)\Gamma\left(\frac{1-2j}{k-2}\right) \frac{\Gamma\left(j+\frac{kw}{2}\right)}{\Gamma\left(1-j+\frac{kw}{2}\right)} \left[e^{i\sigma(1-2j)} + (-)^{kw} e^{-i\sigma(1-2j)}\right] \end{aligned} \tag{5.2}$$

and the closed string annulus amplitude contains only couplings to the continuous representations. However it is clear that further work is needed to obtain a good understanding of this family of D-branes which has important applications, for instance to obtain a CFT description of the Hanany-Witten effect of anomalous creation of D-branes by NS5-branes [32].

To construct the symmetric D-brane of AdS_3 associated with this D2-brane of the cigar we choose Neumann boundary conditions for the extra $\mathrm{U}(1)$. We get then an euclidean H_2 S-brane in lorentzian AdS_3 with profile

$$\cosh r \sin t = \sin\sigma \tag{5.3}$$

and magnetic field (in the first gauge)

$$F = \frac{k}{2\pi} \frac{\sin\sigma \tanh r}{\sqrt{\cosh^2 r - \sin^2\sigma}}\, \mathrm{d}r \wedge \mathrm{d}\varphi \tag{5.4}$$

more precisely, we have two copies of such an S-brane. As for the D-instanton we can focus on the single cover of AdS_3 with $N=1$. The one-point function for the H_2 spacelike D-brane is then given by:

$$\langle V^j_{nw\gamma pW} \rangle \; = \; \frac{k\sqrt{R}}{2\pi(k-2)} \left(\frac{k-2}{2}\right)^{1/4} \nu_k^{1/2-j}\, \delta_{n,0}\, \delta_{n+kp,0} \times \tag{5.5}$$

[9] There are D2-branes covering only part of the cigar at the level of the semi-classical analysis, but their construction in the CFT is not clear, see however [25].

$$\times\Gamma\left(2-2j\right)\Gamma\left(\frac{1-2j}{k-2}\right)\frac{\Gamma\left(j+\frac{kw-\gamma}{2}\right)}{\Gamma\left(1-j+\frac{kw-\gamma}{2}\right)}\left[e^{i\sigma(1-2j)}+(-)^{kw-\gamma}e^{-i\sigma(1-2j)}\right].$$

After computations very similar to those for the AdS_2 D-branes, we obtain the following relative open string partition function:

$$Z^{\mathrm{OP}}_{\mathrm{H}_2}(\tau)=\sum_{n\in\mathbb{Z}}\int_0^\infty dP\left\{\frac{\partial}{2i\pi\partial P}\log\frac{R(P|i\frac{\sigma+\sigma'}{2})}{R(P|i\frac{\sigma_*+\sigma'_*}{2})}\lambda^c_{P,n}(\tau)+\right.$$
$$\left.+\frac{\partial}{2i\pi\partial P}\log\frac{R(P|i\frac{\sigma-\sigma'}{2})}{R(P|i\frac{\sigma_*-\sigma'_*}{2})}\lambda^c_{P,n+\frac{k}{2}}(\tau)\right\}\sum_{\omega\in\mathbb{Z}}q^{\frac{(n+k\omega)^2}{R^2k}}. \quad (5.6)$$

For the lorentzian AdS_3 (i.e. $R^2=-1$) it gives characters of continuous representations of $\mathrm{SL}(2,\mathbb{R})$ as well as their images under spectral flow. Open strings with odd spectral flow number (second line) are stretched between the two copies of the H_2 D-brane. However the interpretation in spacetime of this open string spectrum is not clear since they are localized in the time direction. It should be possible to construct a boundary state describing the decay of this S-brane, by an analogue of the rolling tachyon solution [43, 44].

5.2 Spacetime-filling D-brane

If we consider instead Dirichlet boundary conditions instead for the extra U(1), we obtain a spacetime-filling D2-brane with a gauge field, given by the same expression (5.4). the one-point function is given by:

$$\langle V^j_{nw\gamma pW}\rangle=\frac{\sqrt{\frac{k}{R}}}{2\pi^2[2(k-2)]^{1/4}}\,\delta_{n,0}\,\delta_{NkW+\gamma,0}\;e^{iNpx_0}\nu^{\frac{1}{2}-j}\times \quad (5.7)$$
$$\times\frac{\Gamma(j+\frac{kw}{2})\Gamma(j-\frac{kw}{2})}{\Gamma(2j-1)\Gamma(1-\frac{1-2j}{k})}\frac{e^{i\sigma(1-2j)}\sin\pi(j-\frac{kw}{2})+e^{-i\sigma(1-2j)}\sin\pi(j+\frac{kw}{2})}{\sin\pi(1-2j)\sin\pi\frac{1-2j}{k}}.$$

The open string annulus partition function can be computed in a straightforward way, and leads to the expression (for the relative partition function):

$$Z^{\mathrm{OP}}_{\mathrm{AdS}_3}(\tau)=\int_0^\infty dP\sum_{n\in\mathbb{Z}}\left\{\frac{\partial}{2i\pi\partial P}\log\frac{R(P|i\frac{\sigma+\sigma'}{2})}{R(P|i\frac{\sigma_*+\sigma'_*}{2})}\lambda^c_{P,n}(\tau)+\right.$$
$$\left.+\frac{\partial}{2i\pi\partial P}\log\frac{R(P|i\frac{\sigma-\sigma'}{2})}{R(P|i\frac{\sigma_*-\sigma'_*}{2})}\lambda^c_{P,n+\frac{k}{2}}(\tau)\right\}\int dE\;\frac{q^{-\frac{E^2}{k}}}{\eta(\tau)} \quad (5.8)$$

giving a result very similar to the symmetry-breaking D-branes associated to the AdS_2 branes.

This concludes our analysis of D-branes in AdS_3. We have considered all the D-branes that can be decomposed in the T-dual geometry in terms of D-branes in the elliptic coset $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1)$. It would be interesting to consider the D-branes related to the hyperbolic and parabolic cosets, for instance to construct D-branes in the BTZ black hole.

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A. Characters of $\mathrm{SL}(2,\mathbb{R})$ and $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$

In this section we recall some properties of the characters of $\mathrm{SL}(2,\mathbb{R})$ and their decomposition in terms of coset characters [45] (see also [46]). We follow closely the presentation of [47]. We discuss the *elliptic basis* diagonalizing the action of the timelike current J^3. Indeed the cigar CFT $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ is obtained by gauging the corresponding subgroup of $\mathrm{SL}(2,\mathbb{R})$.

A.1 Continuous representations

The continuous representations of $\mathrm{SL}(2,\mathbb{R})$ are labeled by a continuous spin $j=\frac{1}{2}+iP$ with $P\in\mathbb{R}_+$, and a second label α with $2\alpha\in\mathbb{Z}_2$ for the single cover of $\mathrm{SL}(2,\mathbb{R})$.[10] The characters are just obtained by the free action of the modes of the currents as:

$$\chi^c_{\frac{1}{2}+iP,\alpha}(\tau,\nu)=\mathrm{Tr}_{\mathrm{rep}(\frac{1}{2}+iP,\alpha)}\left(q^{L_0-c/24}e^{2i\pi\nu J^3_0}\right)=\frac{q^{\frac{P^2}{k-2}}}{\eta^3}\sum_{n\in\mathbb{Z}}e^{2i\pi\nu(\alpha+n)}\,. \tag{A.1}$$

The continuous characters of the coset are obtained by decomposing the $\mathrm{SL}(2,\mathbb{R})$ characters according to the $\widehat{\mathfrak{u}(1)}$ affine subalgebra of $\widehat{\mathfrak{sl}(2,\mathbb{R})}$ that is gauged:

$$\chi^c_{\frac{1}{2}+iP,\alpha}(\tau,\nu)=\sum_{r\in\mathbb{Z}}\lambda^c_{\frac{1}{2}+iP,r+\alpha}\,e^{2i\pi\nu(r+\alpha)}\frac{q^{-\frac{(r+\alpha)^2}{k}}}{\eta(\tau)}=\sum_{r\in\mathbb{Z}}q^{\frac{P^2}{k-2}}\frac{q^{\frac{(r+\alpha)^2}{k}}}{\eta^2}\,e^{2i\pi(r+\alpha)}\frac{q^{-\frac{(r+\alpha)^2}{k}}}{\eta(\tau)} \tag{A.2}$$

thus giving the coset characters $\lambda^c_{\frac{1}{2}+iP,r+\alpha}$. Their modular transformation is straightforward and reads:

$$\lambda^c_{\frac{1}{2}+iP,r+\alpha}(-1/\tau)=\frac{4}{\sqrt{k(k-2)}}\int\mathrm{d}\mu\,e^{-4i\pi\frac{(r+\alpha)\mu}{k}}\int_0^\infty\mathrm{d}P'\cos\left(\frac{4\pi PP'}{k-2}\right)\lambda^c_{\frac{1}{2}+iP',\mu}(\tau)\,. \tag{A.3}$$

New characters of $\mathrm{SL}(2,\mathbb{R})$ are obtained by the *spectral flow* external automorphism of the $\widehat{\mathfrak{sl}(2,\mathbb{R})}$ affine algebra [3] of integer parameter w:

$$\tilde{J}^3_n=J^3_n-\frac{kw}{2}\delta_{n,0}\,,\qquad \tilde{J}^\pm_n=J^\pm_{n\pm w}\,. \tag{A.4}$$

This is an external automorphism of the affine algebra generating new representations whose spectrum is not bounded from below. Nevertheless they are necessary to obtain a consistent AdS_3 string theory. The continuous representations with spectral flow corresponds in AdS_3 to "long strings" that are macroscopic fundamental strings that expands

[10] Otherwise $2N\alpha\in\mathbb{Z}_{2N}$ for the N-th cover.

towards the boundary of AdS_3 while winding w times. The corresponding characters can be written using the coset decomposition as:

$$\chi^{c\,(w)}_{\frac{1}{2}+iP,\alpha}(\tau,\nu) = \sum_{r\in\mathbb{Z}} \lambda^{c}_{\frac{1}{2}+iP,r+\alpha}\, e^{2i\pi(r+\alpha+kw/2)\nu}\, \frac{q^{-\frac{(r+\alpha+kw/2)^2}{k}}}{\eta(\tau)}\,. \tag{A.5}$$

A.2 Discrete representations

The lowest weight discrete representations are characterized by a real spin $2j \in \frac{\mathbb{Z}}{N}$ (for the N-th cover) and the charge of the states in a representation j under the elliptic subalgebra is $J^3_0 = j + r$, with $r \in \mathbb{Z}$ (the primary states have $r \in \mathbb{N}$). Their characters are more involved and reads:

$$\chi^d_j(\tau,\nu) = \frac{q^{-\frac{(j-\frac{1}{2})^2}{k-2}}\, e^{2i\pi\nu(j-\frac{1}{2})}}{i\vartheta_1(\tau,\nu)} \tag{A.6}$$

with

$$\vartheta_1(\tau,\nu) = -ie^{-i\pi\nu} q^{3/24} \prod_{n=1}^{\infty} (1-q^n)(1-e^{2i\pi\nu}\, q^{n-1})(1-e^{-2i\pi\nu}\, q^n)\,. \tag{A.7}$$

Then the characters of the coset are obtained by a decomposition similar to (A.2), and reads:

$$\lambda^d_{j,r}(\tau) = \frac{q^{-\frac{(j-\frac{1}{2})^2}{k-2}}}{\eta^2(\tau)} S_r(\tau) q^{\frac{(j+r)^2}{k}} \qquad \text{with} \qquad S_r(\tau) = \sum_{n=0}^{\infty} (-)^n q^{\frac{n(n+2r+1)}{2}}\,. \tag{A.8}$$

The modular transformation of these characters are not known. However, for integer k one can define *extended characters* as $\Lambda^d_{j,n} = \sum_\omega \lambda^d_{j,n+k\omega}$. Their modular transformation has been computed in [48].

For the discrete representations one can also define spectral flowed characters of $SL(2,\mathbb{R})$, that can be written as:

$$\chi^{d\,(w)}_j(\tau,\nu) = \sum_{r\in\mathbb{Z}} \lambda^j_{j,r}\, e^{2i\pi(j+r+kw/2)\nu}\, \frac{q^{-\frac{(j+r+kw/2)^2}{k}}}{\eta(\tau)}\,. \tag{A.9}$$

A.3 Identity representation

The character of $SL(2,\mathbb{R})$ associated to the identity representation reads:

$$\chi^{\mathbb{I}}(\tau,\nu) = -\frac{q^{-\frac{1}{4(k-2)}}\; 2\sin\pi\nu}{\vartheta_1(\tau,\nu)} \tag{A.10}$$

and the only primary state of the affine algebra has $J^3_0 = 0$. It is expanded in terms of coset characters as:

$$\chi^{\mathbb{I}}(\tau,\nu) = \sum_{r\in\mathbb{Z}} \lambda^{\mathbb{I}}_r(\tau)\, e^{2i\pi r}\, \frac{q^{-\frac{r^2}{k}}}{\eta(\tau)} \tag{A.11}$$

where

$$\lambda^{\mathbb{I}}_r(\tau) = \frac{q^{-\frac{1}{4(k-2)}}}{\eta(\tau)^2} q^{|r|+\frac{r^2}{k}} \left[1 + \sum_{n=1}^{\infty} (-)^n q^{\frac{n^2+(2|r|+1)n-2|r|}{2}} (1+q^{|r|})\right]. \tag{A.12}$$

One can define spectral flowed identity characters as well.

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A.4 Fourier transform

The bulk two-point function in the coset $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ has been obtained in [35] by using the following Fourier transform:

$$\Phi^j_{m\bar{m}} = \frac{1}{4\pi^2}\int_{\mathbb{C}} d^2x\, x^{j+m-1}\bar{x}^{j+\bar{m}-1}\,\Phi^j(x,\bar{x}) \tag{A.13}$$

in order to diagonalize the action of J^3. The fields in H_3^+ satisfy the following reflection property [6]:

$$\Phi^j(x,\bar{x}) = R(j)\frac{1-2j}{\pi}\int d^2u|u-x|^{-4j}\,\Phi^{1-j}(u,\bar{u}) \tag{A.14}$$

written in terms of the CFT reflection amplitude

$$R(j) = -\nu^{1-2j}\frac{\Gamma(1+\frac{1-2j}{k-2})}{\Gamma(1-\frac{1-2j}{k-2})} \tag{A.15}$$

and involving the intertwining operator $\mathcal{I}^j$ realizing the isomorphism of the representations of spin j and $1-j$ in SL(2,$\mathbb{C}$), of kernel:

$$I^j(u|x) = \frac{1-2j}{\pi}|u-x|^{-4j} \tag{A.16}$$

normalized such that $\mathcal{I}^{1-j}\circ\mathcal{I}^j = \mathbb{I}$. Then one obtains by the Fourier transform the reflection amplitude in the coset that is used to write eq. (2.17).

Similarlily one can define a Fourier transform for the boundary operators corresponding to AdS_2 branes with boundary conditions $(r\,r')$ by:

$$\Phi^{j(rr')}_m = \int_{\mathbb{R}} dx\, x^{j-1+m}\,\Phi^{j(rr')}(x) \tag{A.17}$$

and in this basis these are eigenfunctions of J^3 with eigenvalue m. The reflection property for such boundary fields reads [20, 40]:

$$\Phi^{j(rr')}(x) = R(j|rr')(\mathcal{J}^j\Phi^{1-j(rr')})(u) \tag{A.18}$$

where the boundary reflection amplitude is:

$$\begin{aligned} R(j|rr') = \nu^{\frac{1}{2}-j}\frac{\Gamma_k^2(k-2+j)}{\Gamma_k^2(k-2+1-j)}\frac{\Gamma_k(k-2+1-2j)}{\Gamma_k(k-2+2j-1)}\frac{S_k^{(0)}\left(\frac{k-2}{2\pi}(r+r')-i(j-1/2)\right)}{S_k^{(0)}\left(\frac{k-2}{2\pi}(r+r')+i(j-1/2)\right)}\times \\ \times\frac{S_k^{(1)}\left(\frac{k-2}{2\pi}(r-r')-i(j-1/2)\right)}{S_k^{(1)}\left(\frac{k-2}{2\pi}(r-r')+i(j-1/2)\right)}\frac{S_k^{(0)}(+i(j-1/2))}{S_k^{(0)}(-i(j-1/2))} \end{aligned} \tag{A.19}$$

and the intertwining operator $\mathcal{J}^j$ realizing the isomorphism of the representations of spin j and $1-j$ in SL(2,$\mathbb{R}$) is also normalized such that $\mathcal{J}^{1-j}\circ\mathcal{J}^j = \mathbb{I}$:

$$(\mathcal{J}^j\Phi^{1-j(rr')})(u) = \frac{1}{\Gamma(1-2j)(1+e^{i\pi(2j-1)})}\int_{\mathbb{R}} dv|u-v|^{-2j}\,\Phi^{1-j(rr')}(v)\,. \tag{A.20}$$

Consequently after a Fourier transform it gives the reflection amplitude given in the text, eq. (3.16).

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